Earth's climate has undergone different intervals of gradual change as well as abrupt shifts between climate states. Here we aim to characterize the corresponding changes in climate response to astronomical forcing in the icehouse portion of the Cenozoic, from the latest Eocene to the present. As a tool, we use a 35-m.y.-long $\delta^{18}O_{benthic}$ record compiled from different high-resolution isotope records spliced together (what we refer to as a megasplice). We analyze the climate response to astronomical forcing during four 800-k.y.-long time windows. During the mid-Miocene Climatic Optimum (ca. 15.5 Ma), global climate variability was mainly dependent on Southern Hemisphere summer insolation, amplified by a dynamic Antarctic ice sheet; 2.5 m.y. later, relatively warm global climate states occurred during maxima in both Southern Hemisphere and Northern Hemisphere summer insolation. At that point, the Antarctic ice sheet grew too big to pulse on the beat of precession, and the Southern Hemisphere lost its overwhelming influence on the global climate state. Likewise, we juxtapose response regimes of the Miocene (ca. 19 Ma) and Oligocene (ca. 25.5 Ma) warming periods. Despite the similarity in $\delta^{18}O_{benthic}$ values and variability, we find different responses to precession forcing. While Miocene warmth occurs during summer insolation maxima in both hemispheres, Oligocene global warmth is consistently triggered when Earth reaches perihelion in the Northern Hemisphere summer. This pattern is in accordance with previously published paleoclimate modeling results, and suggests an amplifying role for Northern Hemisphere sea ice.