New Algorithms to Calculate and Interpret U-Th and U-Th-Pb Dates

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The simple age equation for a single parent-daughter isotope pair becomes more complicated when intermediate daughter isotopes are added. The intermediate daughters in the ^{238}U decay chain are the basis of two important clocks, the $^{230}Th/U$ chronometer for silicate systems, which assumes that ^{234}U is in secular equilibrium with ^{238}U, and the U-Th chronometer for carbonates, which relies on disequilibrium in the ^{238}U-^{234}U-^{230}Th sytem. Existing approaches to calculating dates and uncertainties can be slow and sometimes inaccurate.

For silicate U-series $^{230}Th/U$ and simple ^{230}Th-^{226}Ra systems, isotope ratio measurements are plotted on isochron diagrams, where a suite of isochronous closed-system minerals or whole-rock samples represents a linear array of data with a slope of zero at zero age, unity at secular equilibrium, and at intermediate times, $t = -1/\lambda \log(1\text{-slope})$. Assuming that the age of the sample is non-negative produces a truncated probability density function (pdf) for young ages, and ages near secular equilibrium or analyses with large uncertainties produce highly asymmetric pdfs. Instead of using Monte Carlo methods with thousands to millions of trials to interpret these analyses, it is possible to derive them analytically, then use the their derivatives and antiderivatives to quickly estimate the mean, median, mode (maximum likelihood estimate), and confidence limits. We utilize this speed to drive interactive visualizations inside the open-source, freely accessible software ET_Redux, allowing users to explore and interpret their data and to facilitiate teaching.

For older conventional carbonate U-series dates, or for dates that incorporate U-series disequilibrium and radiogenic Pb ingrowth, an additional concern is the accurate treatment of the full U-series decay chain. This includes the slowly declining ^{238}U activity that is not included in a traditional Bateman equation solution and complex ^{234}U, ^{230}Th, ^{226}Ra and ^{222}Rn disequilibrium in the ^{238}U-^{206}Pb system. We utilize a matrix exponential equation that accurately incorporates all isotopes of interest. This easily differentiable form can be solved quickly with Newton-Raphson, produces slightly different U-series dates at older ages and different isotope ratios at (transient, not secular) equilibrium, and is easy to differentiate to draw smooth vectorized Bezier curves for an interactive U-series evolution plot in ET_Redux.