Ar-Ar_Redux: Rigorous error propagation of $^{40}Ar/^{39}Ar$ data, including covariances

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Let z = f(x,y). Then standard error propagation of z by first order Taylor expansion yields:

$$\sigma_z^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + 2\frac{\partial f}{\partial x}\frac{\partial f}{\partial y} cov(x,y)$$

where cov(x,y) is the 'covariance of x and y'. Current practice in $^{40}Ar/^{39}Ar$ geochronology assumes that the third term of this equation can be safely neglected. For example, consider the $^{40}Ar/^{39}Ar$ age equatin [t=ln(1+RJ)/λ, where λ is the decay constant of ^{40}K, R is the $^{40}Ar^*/^{39}Ar_K$, ratio and J is the irradiation parameter]. Then the age uncertainty is currently calculated as:

$$\sigma_t^2 = \frac{J^2\sigma_R^2 + R^2\sigma_J^2}{\lambda^2(1+RJ)}$$

which assumes that cov(R,J) = 0. This assumption cannot be correct because both R and J are calculated using the same mass fractionation corrections, detector calibrations, interference corrections and radioactive decay corrections. The analytical uncertainty associated with each of these factors results in correlated errors between R and J. Ignoring these error correlations affects both the precision and accuracy of the resulting $^{40}Ar/^{39}Ar$ ages. The problem of correlated errors crops up everywhere in the $^{40}Ar/^{39}Ar$ method. In fact, the covariant structure is deeply engrained into the very DNA of the method, which is based on five isotopes (36-40) of a single element (Ar). These isotopes are subjected to a constant sum constraint which makes them correlated by definition. Correlated errors are created during mass spectrometry, when the ion detector signals are extrapolated to 'time zero' and blank corrections are made. They occur as a result of mass fractionation corrections and detector inter-calibrations. They arise when accounting for the effect of radioactive decay on ^{39}Ar and ^{37}Ar, or whenever an interference correction is made. Error correlations occur when calculating J-factors to solve the age equation (see above), and when calculating the weighted mean of several $^{40}Ar/^{39}Ar$ age analyses. Finally, error correlations are associated with the the systematic biases that occur as a result of the uncertainty in the ^{40}K decay constant and the atmospheric $^{40}Ar/^{36}Ar$ ratio. Thus, the existence of correlated errors affects every aspect of the $^{40}Ar/^{39}Ar$ method. Ar-Ar_Redux is a new and fully functional 'one stop' computer code for $^{40}Ar/^{39}Ar$ data reduction which propagates all the analytical uncertainties in matrix form and thus keeps track of all error correlations.