

# Geochemical Evolution of an Heterogeneous, Open and Convective Reservoir: A Theoretical Approach

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Geochemical information about the Earth mantle relies on samples collected at the Earth surface, with little memory of the location where they acquired their geochemical characteristics. Therefore, the numerical approach that excels for geophysical modelling of the convective Earth mantle is not appropriate. Consequently, the classical approach in geochemistry is based on the concept of reservoir, a synonym of "system" used in thermodynamics, with the restriction that the extensive parameters are quite solely chemical species abundances (or masses), which is described statistically rather than geographically.

Thus to fully describe the state of the reservoir, one must use the distribution of the different chemical species relative to the abundance of a reference one, or to the mass of implied matter. Mathematically, this translates into the use of the statistical tool of multivariate probability density functions, or, as called by the mathematician L.Schwarz, a "distribution" per se. A coarser description will only rely of the bulk values of the different (intensive) geochemical parameters. An intermediate description will need some macroscopic measure of the reservoir heterogeneity. Owing for some light hypothesis, we will show how bulk values are linked to the statistical mean values of the distribution, not a surprise, and how heterogeneity can be described by the standard deviations.

In order to describe the evolution of the reservoir when subject to input and output fluxes, or to convective homogenization, ones uses the formalism of budget constraints. We will develop this for the reservoir distribution, and shows its coherency with the classical evolution equations for the bulk

geochemical parameters. In particular, we will emphasize on the relationships between the different time concepts : residence times, characteristic times for the fluxes, average age of the reservoir, etc., and show how these relationships are sensitive to the steady-state hypothesis.

The next step will be to deduce the evolution equation of the heterogeneity for a geochemical parameter, more precisely of its variance, square of the standard deviation. The important point is that the term relative to convective homogenization does not vanish, which permits to deepen the concept of stirring time. Therefore, assuming a 2nd order steady state (steady state of heterogeneity, while bulk parameters may still evolve), we will deduce simple relationships between the different residence times and this stirring time. However, we will show, on the base of numerical convective simulations, that the exponential law classically used to describe convective homogenization, is not necessarily adequate.

Finally, we will consider the covariance of a pair of geochemical parameters, in the specific case of an isotopic ratio and its associated parent-daughter chemical ratio. This will permits to come back to the relationship between model age, stirring time and residence time that was proposed by Allègre and Lewin, 1995, based on empiric equations for the heterogeneity evolution.

Allègre, CJ & Lewin, É, *Earth Planet. Sci. Lett.*, **136**, 629-646, (1995).